A T_0-discrete universe model with five low-energy fundamental interactions and the coupling constants hierarchy

Vladimir N. Efremov*
Mathematics Department, CUCEI, University of Guadalajara, Guadalajara, Jalisco, Mexico.

Nikolai V. Mitskievich†
Physics Department, CUCEI, University of Guadalajara, Guadalajara, Jalisco, Mexico.

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A quantum model of universe is constructed in which values of dimensionless coupling constants of the fundamental interactions (including the cosmological constant) are determined via certain topological invariants of manifolds forming finite ensembles of 3D Seifert fibrations. The characteristic values of the coupling constants are explicitly calculated as the set of rational numbers (up to the factor 2π) on the basis of a hypothesis that these values are proportional to the mean relative fluctuations of discrete volumes of manifolds in these ensembles. The discrete volumes are calculated using the standard Alexandroff procedure of constructing T_0-discrete spaces realized as nerves corresponding to characteristic canonical triangulations which are compatible with the Milnor representation of Seifert fibered homology spheres being the building material of all used 3D manifolds. Moreover, the determination of all involved homology spheres is based on the first nine prime numbers ($p_1 = 2, \ldots, p_9 = 23$). The obtained hierarchy of coupling constants at the present evolution stage of universe well reproduces the actual hierarchy of the experimentally observed dimensionless low-energy coupling constants.

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Almost half a century ago Wheeler [1] has put forward the idea of the quantum spacetime topology fluctuations. This idea had been further developed by Hawking [2] using the path integral techniques in the Euclidean quantum gravity. Topology fluctuations are still a crucial element of the wormhole (or, baby-universe) mechanism for vanishing of the cosmological constant [3]. Some efforts were also made to show that the entire effect of the spacetime topology fluctuations were to modify all fundamental coupling constants in physics and to provide a probability distribution for them (see, *e.g.*, [4]). We develop here similar ideas, but now in terms of the T_0-discrete spacetime realized as the inverse spectrum of nerves of the sequence of triangulations [5] (*a discrete canonical approach*). This makes it possible to quantitatively deduce the hierarchy of *dimensionless low-energy coupling* (DLEC) constants of the known fundamental interactions, as well as the evolution of these “constants” while the universe goes through a sequence of “inflationary” stages. To this end we use a finite ensemble of topological spaces $\mathcal{M}$ which describes a spectrum of the topology fluctuations of three-dimensional spatial sections. Just the numerical characteristics of these fluctuations determine magnitudes of the coupling constants. The principal constructive elements in building the ensemble $\mathcal{M}$ are the *Seifert fibered homology spheres* (Sfh-spheres) characterized by collections of three mutually prime numbers. Moreover, the determination of all involved Sfh-spheres is based on the first nine prime numbers ($p_1 = 2, \ldots, p_9 = 23$). So one can trace a relation between the coupling constants hierarchy of the fundamental interactions and the sequence of the prime numbers in the beginning of the set of positive integers $\mathbb{N}$.

T_0-discrete spaces

By an Alexandroff space [6] we mean a topological space every point of which has a minimal neighborhood (assignment of minimal neighborhoods of all points fixes the topology of every Alexandroff's space). We shall consider here only the Alexandroff spaces with the T_0 separability axiom which we call T_0-discrete spaces. Nerves of triangulations of smooth compact manifolds represent an important example of the T_0-discrete spaces [6]. Recall that any smooth compact three-dimensional manifold X admits a *finite closed partition* $\tau = \{T_1, \ldots, T_N\}$ consisting of closed 3D-simplices (tetrahedra) T_i such that $\dim(T_i \cap T_j) < 3$, for $\forall\, i, j = 1, \ldots, N,\ i \neq j$. This partition is called the (*canonical*) c-triangulation. We call the *nerve* of a c-triangulation τ the topological space (simplicial complex) X_τ whose points (simplices) are collections $\{T_{i_0}, \ldots, T_{i_q}\} =: x_{i_0 \ldots i_q}$ of elements of τ for which $T_{i_0} \cap \cdots \cap T_{i_q} \neq \emptyset$, while the minimal neighborhood of any point $x_{i_0 \ldots i_q}$ is defined as the set of points $x_{j_0 \ldots j_p} \in X_\tau$ such that $T_{j_0} \cap \cdots \cap T_{j_p} \subseteq T_{i_0} \cap \cdots \cap T_{i_q}$. Any nerve is a T_0-discrete space [6]. In the topology defined by the last relation, not all points of the nerve are closed since X_τ is *not* a Hausdorff space. The (*closed*) c-points of X_τ are those and only those which correspond to tetrahedra, *i.e.* $x_{i_0} = \{T_{i_0}\}$. Later on we shall call *discrete volume* of a manifold X, corresponding to c-triangulation, the

*Electronic address: efremov@udgserv.cencar.udg.mx
†Electronic address: nmitskie@udgserv.cencar.udg.mx

number N of tetrahedra in the triangulation τ, or equivalently, the number of c-points in the nerve X_τ.

Seifert fibered homology spheres

The basic structural elements of which we construct our cosmological model, are Sfh-spheres. We shall make use of explicit analytic description of Sfh-spheres with three exceptional fibers [7]. Let $\Sigma(a_1, a_2, a_3) =: \Sigma(\underline{a})$ be the smooth compact three manifold obtained by intersecting the complex algebraic Brieskorn surface $z_1{}^{a_1} + z_2{}^{a_2} + z_3{}^{a_3} = 0$ $(z_i \in \mathbb{C}_i)$ with the unit five dimensional sphere $|z_1|^2 + |z_2|^2 + |z_3|^2 = 1$, where a_1, a_2, a_3 are pairwise coprime integers, $a_i > 1$. There exists a unique Seifert fibration of this manifold with unnormalized Seifert invariants [8]: (a_i, b_i) subject to $e(\Sigma(\underline{a})) = \sum_{i=1}^{3} b_i/a_i = 1/a$, where $a = a_1 a_2 a_3$ and $e(\Sigma(\underline{a}))$ is the well known topological invariant of a Sfh-sphere also called its Euler number.

To construct our model of universe we need a specific family of Sfh-spheres which would be defined in following way. First, the derivative of a Sfh-sphere $\Sigma(\underline{a}) := \Sigma(a_1, a_2, a_3)$ can be defined as a Sfh-sphere

$$\Sigma^{(1)}(\underline{a}) := \Sigma(a_1, a_2 a_3, a+1) \equiv \Sigma(a_1^{(1)}, a_2^{(1)}, a_3^{(1)}). \quad (1)$$

The Euler number of this Sfh-sphere is $e(\Sigma^{(1)}(\underline{a})) = 1/a^{(1)}$ where $a^{(1)} = a_1^{(1)} a_2^{(1)} a_3^{(1)} = a(a+1)$. By induction, we define the derivative $\Sigma^{(l)}(\underline{a}) = \Sigma(a_1^{(l)}, a_2^{(l)}, a_3^{(l)})$ of $\Sigma(\underline{a})$ of any order l. In particular, there holds the recurrent relation

$$a^{(l)} = a^{(l-1)} \left(a^{(l-1)} + 1 \right) \quad (2)$$

for a product of three Seifert invariants $a^{(l)} = a_1^{(l)} a_2^{(l)} a_3^{(l)}$. Second, we define a sequence of Sfh-spheres which we shall call *primary sequence*. Let p_i be a prime number being the ith in the set of the positive integers $\mathbb{N}$, *e.g.*, $p_1 = 2,\ p_2 = 3, \ldots$. The primary sequence of Sfh-spheres is defined as

$$\{\Sigma(q_i, p_{i+1}, p_{i+2}) | i \in \mathbb{N}\} \quad (3)$$

where $q_i = p_1 \cdots p_i$. Finally, to the end of constructing our model of universe, we include in this sequence as its first two terms the usual *three-dimensional spheres* S^3 *with Seifert's fibrations* (Sf-spheres) determined by the mappings $h_{pq} : S^3 \to S^2$ [9]. Recall that $S^3 = \left\{ (z_1, z_2) \,\middle|\, |z_1|^2 + |z_2|^2 = 1 \right\}$ and $z_1^p / z_2^q \in \mathbb{C} \cup \{\infty\} \cong S^2$. We denote these two Sf-spheres as $\Sigma(1, 1, 2)$, $p = 1, q = 2$ and $\Sigma(1, 2, 3)$, $p = 2, q = 3$. In these notations we use an additional third number (unit) which corresponds to an arbitrary regular fiber. This will enable us to take derivatives of Seifert fibrations on $\Sigma(1, 1, 2)$ and $\Sigma(1, 2, 3)$ by the same rule (1) as for other members of the sequence (3).

Now we form the family of manifolds corresponding to the first primary Sfh-spheres and their derivatives up to the fourth order,

$$\{\Sigma^{(l)}(q_{i-1}, p_i, p_{i+1}) | i \in \overline{0,8}, l \in \overline{0,4}\}, \quad (4)$$

where $\overline{0,n}$ is the integer numbers interval from 0 to n. Note that the subfamily corresponding to $i = 0, 1$ is built of the ordinary spheres S^3 with fixed Seifert fibrations. In order to include the Sf-spheres $\Sigma(1, 1, 2)$ and $\Sigma(1, 2, 3)$ in this family, one has to put $q_{-1} = q_0 = p_0 = 1$. *E.g.*, for the well known Poincaré homology sphere $\Sigma(p_1, p_2, p_3) = \Sigma(2, 3, 5)$, the sequence of derivatives is

$$\Sigma^{(1)}(2,3,5) = \Sigma(2, 15, 31),$$
$$\Sigma^{(2)}(2,3,5) = \Sigma(2, 465, 931),$$
$$\Sigma^{(3)}(2,3,5) = \Sigma(2, 432915, 865831),$$
$$\Sigma^{(4)}(2,3,5) = \Sigma(2, 374831227365, 749662454731).$$

Having done calculations of the Euler numbers of Seifert structures of Sf- and Sfh-spheres in the family (4), we find that for the subfamily

$$\left\{ \Sigma^{(l)} = \Sigma^{(l)}(q_{2l-1}, p_{2l}, p_{2l+1}) \,\middle|\, l \in \overline{0,4} \right\} \quad (5)$$

the Euler numbers (multiplied by 2π) reproduce fairly well the real hierarchy of DLEC constants of fundamental interactions, see Table I.

TABLE I: Euler numbers *vs.* experimental DLEC constants.

l	$e\left(\Sigma^l\right)$	Interaction	$\alpha_{\text{exper}}{}^a$
0	3.14	strong	1
1	6.78×10^{-3}	electromagnetic	7.20×10^{-3}
2	2.20×10^{-13}	weak	3.04×10^{-12}
3	1.36×10^{-45}	gravitational^b	2.73×10^{-46}
4	1.67×10^{-133}	cosmological	$< 10^{-120}$

[a]Reference [11].
[b]Normalized with respect to the electron mass.

The agreement of the calculated hierarchy of the DLEC constants and the experimental data suggests the idea to construct a model of universe glued up of Sf- and Sfh-spheres using the connected sum operation (compact locally homogeneous universes with spatial sections homeomorphic to Seifert fibrations were considered in [10]). To this end we primarily have to reduce and reparametrize the family (4). First, in accordance with (5), we eliminate the Sf- and Sfh-spheres with odd numbers i introducing a new parameter $n \in \overline{0,4}$ related to i as $i = 2n$. Then (in certain cases) it is convenient also to use another parameter $t = n - l,\ t \in \overline{-4,4}$. The resulting family of Sf- and Sfh-spheres is

$$\left\{ \Sigma_n^{(l)} := \Sigma^{(n-t)}(q_{2n-1}, p_{2n}, p_{2n+1}) \,\middle|\, n \in \overline{0,4}, t \in \overline{-4,4} \right\}, \quad (6)$$

which contains (5) as a subset for $t = 0$, *i.e.* when $n = l$. Parameter t in our model is the discrete cosmological "time", $t = 0$ labelling the present state of the universe where an observer can determine the DLEC constants $\alpha_n^{(n)}$ of the five $(n \in \overline{0,4})$ fundamental interactions (see Table I). The relation (2) readily yields good estimates of the DLEC constants $\alpha_n^{(n)} = 2\pi e \left(\Sigma_n^{(n)} \right) \simeq 2\pi (q_{2n+1})^{-2^n}$.

Remember that $q_{2n+1} = p_1 \cdots p_{2n+1}$ is product of the first $2n+1$ prime numbers in $\mathbb{N}$. Note that for $n=5$ there would be $\alpha_5^{(5)} \simeq 1.4 \cdot 10^{-357}$ which is too small to be identified with a certain experimentally determined coupling constant, thus we put $n, l \in \overline{0,4}$.

Though this approach leads to a hierarchy of the DLEC constants, it yields neither a description of the evolution of universe, nor other its features, therefore we pass to framing a more constructive universe model glued of Sf- and Sfh-spheres in accordance with the method described in [5] a particular case of which we present below. To this end we first have to calculate the discrete volumes of Sf- and Sfh-spheres from the family (6).

Canonical characteristic triangulations

We now have to find such minimal c-triangulations of a Sfh-sphere $\Sigma(a_1, a_2, a_3)$ which are compatible with its Milnor representation as a a_3-fold cyclic branched covering of S^3 branched along a torus knot $K(a_1, a_2)$ [7]. We call this triangulation the *characteristic canonical triangulation* (chc-triangulation or τ_c). The formula giving the number of tetrahedra in τ_c, or the number of c-points in the corresponding nerve, is invariant under permutations of Seifert invariants a_1, a_2, a_3 since the Milnor representation is symmetric under their permutations.

A c-triangulation is said to be compatible with the Milnor representation of $\Sigma(a_1, a_2, a_3)$ if the knot $K(a_1, a_2)$ consists of one-dimensional simplices of the triangulation induced by τ_c. To find the chc-triangulation, consider in S^3 a torus T^2 with a torus knot $K(a_1, a_2)$ on it. The simplest triangulation of this torus when the knot $K(a_1, a_2)$ is a 1-cycle, contains $2a_1a_2$ triangles. These triangles are now used as 2-faces in c-triangulation of the sphere S^3 divided by T^2 into two solid tori ST_1 and ST_2. This triangulation of the torus T^2 induces the minimal triangulation of each of the two solid tori, which consists of $8a_1a_2$ tetrahedra. Thus the total number of tetrahedra in the c-triangulation of S^3 is $16a_1a_2$. Lifting this triangulation to $\Sigma(a_1, a_2, a_3)$, we get $16a_1a_2a_3 = 16a$ tetrahedra in the chc-triangulation.

Note that the Milnor covering $\pi : \Sigma(a_1, a_2, a_3) \to S^3$ is a simplicial one [6] in the sense that chc-triangulation of $\Sigma(a_1, a_2, a_3)$ with the number of tetrahedra $16a$ is a covering one of either of c-triangulations of S^3 with the numbers of tetrahedra $16a_1a_2$, or $16a_1a_3$, or $16a_2a_3$; we shall call them chc-triangulations of S^3. Therefore when at the nth level the gluing is performed of either $\Sigma(a_1, a_2, a_3)$ or S^3, $\Sigma(a_1, a_2, a_3)$ is taken in chc-triangulation with $N(\Sigma) = 16a$ tetrahedra, while S^3, in any one of its three chc-triangulations, so that the mean number of tetrahedra now is $N(S) = (16/3)(a_1a_2 + a_1a_3 + a_2a_3)$. Here $N(\Sigma)$ and $N(S)$ are measures of discrete volume of the manifolds $\Sigma(a_1, a_2, a_3)$ and S^3 respectively. Note that at the nth level $\Sigma(a_1, a_2, a_3) = \Sigma^{(l)}(q_{2n-1}, p_{2n}, p_{2n+1})$. Numbers of tetrahedra in the nth-level triangulations are written as $N_n^{(l)}(\Sigma)$ and $N_n^{(l)}(S)$ (the gluing is performed under the condition $l = \text{const}$).

Universe with one fundamental interaction

In our model every fundamental interaction will be characterized by a pair of discrete parameters (n, l), where $n, l \in \overline{0,4}$. Any (n, l)-interaction is related to an ensemble $\mathcal{M}_n^{(l)}$ of topological spaces $M_n^{(l)}(R)$ which are interpreted as a set of admissible spatial sections of a "universe" involving only one fundamental interaction. Such a "universe" will be called (n, l)-universe.

Let the ensemble $\mathcal{M}_0^{(l)}$ consist of merely one manifold $M_0^{(l)} = \Sigma_0^{(l)}$, a Sf-sphere from the family (6). The chc-triangulation of this Sf-sphere contains $N_0^{(l)}(\Sigma)$ tetrahedra, thus the discrete volume of $M_0^{(l)}$ is equal to $V_0^{(l)} = N_0^{(l)}(\Sigma)$. The ensemble $\mathcal{M}_1^{(l)} := \left\{ M_1^{(l)}(R) \middle| R \in \overline{0, V_0^{(l)}} \right\}$ consists of components $M_1^{(l)}(R)$ every one of which is obtained from the manifold $M_0^{(l)}$ by R-fold application of the connected sum operation involving the Sfh-sphere $\Sigma_1^{(l)}$ and $\left(V_0^{(l)} - R\right)$-fold application of the same operation involving S^3. Strictly speaking, one has to remove R tetrahedra from the manifold $M_0^{(l)}$ and to attach instead of them R Sfh-spheres $\Sigma_1^{(l)}$ (from each of the latters one tetrahedron is also supposed to be removed). A similar procedure has to be performed using $\left(V_0^{(l)} - R\right)$ chc-triangulated spheres. The obtained manifold $M_1^{(l)}(R)$ has discrete volume (tetrahedra number in the resulting triangulation)

$$N_1^{(l)}(R) = R N_1'^{(l)}(\Sigma) + \left(V_0^{(l)} - R\right) N_1'^{(l)}(S) \qquad (7)$$

where $N_1'^{(l)}(\Sigma) = N_1^{(l)}(\Sigma) - 1$ and $N_1'^{(l)}(S) = N_1^{(l)}(S) - 1$; (-1) appeared here due to the removal of a tetrahedron from both $\Sigma_1^{(l)}$ and S^3 when the connected sum was applied. Note that the discrete volume $N_1^{(l)}(R)$ does not depend on the choice of concrete R tetrahedra instead of which the Sfh-spheres are pasted. Let us suppose that the probabilities of pasting an Sfh-sphere and the usual S^3, are the same ($p_\Sigma = p_S = 1/2$), and characterize the ensemble $\mathcal{M}_1^{(l)}$ by the *mean discrete volume* (md-volume) of the $(1, l)$-universe $V_1^{(l)} = [\sum_{R=0}^{V_0^{(l)}} w_1^{(l)}(R) N_1^{(l)}(R)]$ where $[\ldots]$ is the integer part of of the number in these brackets, while $w_1^{(l)}(R) = \binom{V_0^{(l)}}{R} / 2^{V_0^{(l)}}$ is a particular case of the well known binomial (Bernoulli) distribution which we relate to the $(1, l)$-interaction.

By induction, the md-volume $V_n^{(l)}$ of the (n, l)-universe can be found from $V_{n-1}^{(l)}$ (md-volume of the ensemble $\mathcal{M}_{n-1}^{(l)} := \left\{ M_{n-1}^{(l)}(R) \middle| R \in \overline{0, V_{n-2}^{(l)}} \right\}$). It reads as

$$V_n^{(l)} = \left[\sum_{R=0}^{V_{n-1}^{(l)}} w_n^{(l)}(R) N_n^{(l)}(R) \right]$$

TABLE II: Mean discrete volumes of (n,l)-universes.

$n \diagdown l$	4	3	2	1	0	-1	-2	-3	-4
0					3.2×10^{1}	9.6×10^{1}	6.7×10^{2}	2.9×10^{4}	5.2×10^{7}
1				1.0×10^{4}	8.6×10^{5}	5.4×10^{9}	2.0×10^{17}	2.7×10^{32}	
2			2.1×10^{8}	3.7×10^{13}	1.3×10^{24}	1.3×10^{45}	1.5×10^{87}		
3		8.9×10^{14}	7.8×10^{25}	6.8×10^{47}	4.9×10^{91}	2.5×10^{179}			
4	1.6×10^{24}	3.1×10^{42}	1.3×10^{82}	2.4×10^{159}	7.5×10^{313}				

where $w_n^{(l)}(R) = \binom{V_{n-1}^{(l)}}{R}/2^{V_{n-1}^{(l)}}$ is the binomial distribution of the (n,l)-interaction which for large $V_{n-1}^{(l)}$ tends to the Gaussian distribution. From the properties of this distribution it follows that

$$V_n^{(l)} = V_{n-1}^{(l)}\left[\left(N_n'^{(l)}(\Sigma) + N_n'^{(l)}(S)\right)/2\right]. \qquad (8)$$

The numerical results for the md-volumes of all (n,l)-universes are given in Table II (up to the second significant digit). Then one can calculate the Shannon entropy $S_n^{(l)} = -\sum_{R=0}^{V_{n-1}^{(l)}} w_n^{(l)}(R)\ln w_n^{(l)}(R)$ of the (n,l)-universe (and in a certain sense of the fundamental interaction of (n,l)-type). The corresponding calculations and discussion of results concerning the cosmological evolution, will be published elsewhere.

We came to the hypothesis that even a "universe" with merely one interaction should be characterized by an ensemble of topological spaces (topologies) $\mathcal{M}_n^{(l)} := \left\{ M_n^{(l)}(R) \middle| R \in \overline{0, V_{n-1}^{(l)}} \right\}$. This mixed-state representation makes the quantum-mechanical description more adequate; moreover, in a universe with several fundamental interactions the quantum-theoretical approach becomes imperative. Let us associate the spacelike section topology $M_n^{(l)}(R)$ with the state vector $|n,l,R\rangle$ characterized by "quantum numbers" $n,l \in \overline{0,4}$, $R \in \overline{0, V_{n-1}^{(l)}}$. Then to the ensemble $\mathcal{M}_n^{(l)}$ corresponds a collection of pure states (basis) $B_n^{(l)}(R) = \left\{ |n,l,R\rangle \middle| R \in \overline{0, V_{n-1}^{(l)}} \right\}$ generating the Hilbert space $\mathcal{H}_n^{(l)}$. Transition between state vectors is realized by operators of creation and annihilation of Sfh-spheres $\Sigma_n^{(l)}$, which we define using the ordinary (boson) commutation relations and the following set of rules:

$$\begin{array}{l} a_n^{(l)+}|n,l,R\rangle = \sqrt{R+1}|n,l,R+1\rangle,\ R < V_{n-1}^{(l)}; \\ a_n^{(l)-}|n,l,R\rangle = \sqrt{R}|n,l,R-1\rangle,\ R > 0; \\ a_n^{(l)+}|n,l,V_{n-1}^{(l)}\rangle = |n+1,l,0\rangle; \\ a_n^{(l)-}|n,l,0\rangle = |n-1,l,V_{n-2}^{(l)}\rangle; \\ a_{n'}^{(l)\pm}|n,l,R\rangle = 0 = a_n^{(l')\pm}|n,l,R\rangle,\ n \neq n',\ l \neq l'. \end{array} \qquad (9)$$

Observation 1. Here, the first two relations show that the operators $a_n^{(l)\pm}$ describe minimal (in our model) topology changes of spatial sections of (n,l)-universe related to joining ($a_n^{(l)+}$) and detaching ($a_n^{(l)-}$) a wormhole with spatial section homeomorphic to the Sfh-sphere $\Sigma_n^{(l)}$. Therefore these operators can be as well interpreted as discrete time shift generators. Topology changes involving Sfh-spheres are, mathematically speaking, Siebenmann-type cobordisms [12], and they were considered from the physical viewpoint in [13].

Observation 2. The state vector $|n,l,0\rangle$ describes the vacuum state of the (n,l)-universe. It is associated with the spatial section $M_n^{(l)}(0)$ which does not contain Sfh-spheres $\Sigma_n^{(l)}$, *i.e.* all $V_{n-1}^{(l)}$ tetrahedra of the $(n-1)$st level chc-triangulation are substituted by ordinary spheres S^3, but with chc-triangulation of the nth level. Therefore by the action of annihilation operator of $\Sigma_n^{(l)}$ on this state (the fourth relation in (9)) occurs a transition to "the nearest lower" state $|n-1,l,V_{n-2}^{(l)}\rangle$ now corresponding to another, $(n-1,l)$, universe.

Observation 3. Concerning the third relation in (9), note that any three-dimensional manifold M is homeomorphic to its connected sum with S^3, *i.e.* $M \cong M\#S^3$. Hence the transition from $|n,l,V_{n-1}^{(l)}\rangle$ to $|n+1,l,0\rangle$ due to the operator $a_n^{(l)+}$, does not represent a topology change, but leads to a sharp growth of the tetrahedra number in the manifold $M_n^{(l)}(V_{n-1}^{(l)})$ chc-triangulation. In fact this is a transition to the manifold $M_{n+1}^{(l)}(0)$ homeomorphic to $M_n^{(l)}(V_{n-1}^{(l)})$, but having considerably more tetrahedra in its chc-triangulation. We relate this phenomenon to the cosmological inflation since in terms of the T_0-discrete spaces (nerves of chc-triangulations) this leads to a rapid growth of the c-points number. For example, for "cosmological" interactions, *i.e.* $(n,4)$-interactions, there are four inflationary stages. This sequence of stages is described by a diagram of transitions between the ensembles $\mathcal{M}_0^{(4)} \xrightarrow{10^{24}} \mathcal{M}_1^{(4)} \xrightarrow{10^{53}} \mathcal{M}_2^{(4)} \xrightarrow{10^{89}} \mathcal{M}_3^{(4)} \xrightarrow{10^{128}} \mathcal{M}_4^{(4)}$ where numbers over the arrows give orders of magnitude of the growth of the tetrahedra number by the transition from the manifold $M_n^{(l)}(V_{n-1}^{(l)})$ of the ensemble $\mathcal{M}_n^{(4)}$ to the manifold $M_{n+1}^{(l)}(0)$ of $\mathcal{M}_{n+1}^{(4)}$. We associate the "ordinary" inflation with the last transition characterized by the 10^{128}-fold discrete volume growth.

According to (9), the pure state $|n,l,R\rangle$ of the (n,l)-universe be an eigenvector of the operator of the nth-level Sfh-spheres number, $a_n^{(l)+}a_n^{(l)-}|n,l,R\rangle = R|n,l,R\rangle$. It is convenient to represent the mixed states of the (n,l)-universe by the density matrix

$$\hat{\rho}_n^{(l)} = \sum_{R=0}^{V_n^{(l)}} w_n^{(l)}(R)|n,l,R\rangle\langle n,l,R|. \qquad (10)$$

The most important observable of the (n,l)-universe is

TABLE III: Dimensionless constants of fundamental interactions

$r_n^{(n-t)}$:	1.4×10^{-35}	9.4×10^{-31}	4.3×10^{-21}	8.8×10^{-2}	3.7×10^{37}	8.9×10^{3}	7.8×10^{-20}	1.2×10^{-33}	1.1×10^{-39}
$n \diagdown t$	4	3	2	1	0	-1	-2	-3	-4
0					1.1×10^{0}	6.4×10^{-1}	2.4×10^{-1}	3.7×10^{-2}	8.7×10^{-4}
1				6.2×10^{-2}	6.8×10^{-3}	8.5×10^{-5}	1.4×10^{-8}	3.8×10^{-16}	
2			4.6×10^{-4}	1.0×10^{-6}	5.6×10^{-12}	1.7×10^{-22}	1.6×10^{-43}		
3		2.1×10^{-7}	7.1×10^{-13}	7.6×10^{-24}	9.0×10^{-46}	1.3×10^{-89}			
4	4.9×10^{-12}	1.1×10^{-21}	5.4×10^{-41}	1.3×10^{-79}	7.3×10^{-157}				

represented by the operator of tetrahedra (c-points) number for the chc-triangulation of a spatial section $M_n^{(l)}(R)$, $\hat{N}_n^{(l)} = V_{n-1}^{(l)} N_n^{(l)}(S) + \left(N_n'^{(l)}(\Sigma) - N_n'^{(l)}(S)\right) a_n^{(l)+} a_n^{(l)-}$, where $V_{n-1}^{(l)} N^{(l)}(S)$ is tetrahedra number in the "vacuum" state $|n,l,0\rangle$ of the (n,l)-universe (see Observation 2). The expectation value of this operator in the mixed state (10) is $\langle N_n^{(l)}\rangle = \mathrm{tr}\left(\hat{\rho}_n^{(l)} \hat{N}_n^{(l)}\right)$. Its integer part is equal to the md-volume $V_n^{(l)}$ of the ensemble $\mathcal{M}_n^{(l)}$ (see (8)). Since this quantity was obtained as a mean value for all admissible topologies of spatial sections of the (n,l)-universe, it should be treated as the complexity characteristic of the T_0-discrete spacetime (the definition see in [5]) where only one fundamental interaction is involved. In the equilibrium (highest probability) state of the (n,l)-universe the observable $N_n^{(l)}$ takes the value $\simeq V_n^{(l)}$ with the standard relative deviation (mean relative fluctuation) $\left\{\langle\left(N_n^{(l)} - \langle N_n^{(l)}\rangle\right)^2\rangle\right\}^{1/2} / \langle N_n^{(l)}\rangle \simeq 1/\sqrt{V_n^{(l)}}$ with respect to the binomial distribution $w_n^{(l)}(R)$. This quantity (multiplied by 2π) we interpret as the dimensionless coupling constant $\alpha_n^{(l)} := 2\pi/\sqrt{V_n^{(l)}}$ of the unique (n,l)-interaction involved in the most probable state of the (n,l)-universe whose spacetime is built of the ensemble $\mathcal{M}_n^{(l)}$ of the spatial sections topologies. Thus in our model the values of coupling constants are determined by the topology fluctuations in the framework of certain finite ensembles. This hypothesis is supported by the fact that the cosmological constant in the De Sitter solution (which is used in inflationary models) is inverse proportional to the square root of the four-dimensional universe volume [14]. Note that $V_n^{(l)}$ is the measure of complexity related just to the spacetime structure of the (n,l)-universe, not to its single spatial section. Hence any coupling constant $\alpha_n^{(l)}$ should be an analogue of the cosmological constant in its proper (n,l)-universe. The values of $\alpha_n^{(l)}$ are given in Table III.

Universes with several fundamental interactions

From the Table III one can see that at $t \equiv n - l = 0$ the values of $\alpha_n^{(l)}$ well represent the hierarchy of the DLEC constants (*cf.* the experimental section of the Table I). Thus the family of ensembles of topologies $\mathcal{M}(0) = \left\{\mathcal{M}_n^{(l)} | n - l = 0\right\}$ should correspond to the present state of the universe with five ($n \in \overline{0,4}$) low-energy fundamental interactions. To describe the states of this compound universe, one should use (as this is usually done treating compound systems) vectors of the Hilbert space $\mathcal{H}(0) := \bigotimes_{n-l=0} \mathcal{H}_n^{(l)}$, the tensor product of all Hilbert spaces related to the (n,l)-universes under the restriction $t = 0$. For other values of $t \in \overline{-4,4}$, the Hilbert space $\mathcal{H}(t) := \bigotimes_{n-l=t} \mathcal{H}_n^{(l)}$ includes the universe states both of the "past" ($t < 0$) and of the "future" ($t > 0$). Note that due to the restriction $n - l \equiv t = \mathrm{const}$, the creation and annihilation operators do *not* bring the state vectors from one Hilbert space to another, if the both are distinct factors in the tensor product of $\mathcal{H}(t)$; however, $\alpha_n^{(l)\pm}$ perform mapping some state vectors from $\mathcal{H}(t)$ to $\mathcal{H}(t \pm 1)$, see the Observations 2 and 3.

From the Table III it follows that the number of (n,l)-interactions in the families $\mathcal{M}(t) = \left\{\mathcal{M}_n^{(l)} | n - l \equiv t = \mathrm{const}\right\}$ is growing from 1 to 5 when t changes from -4 to 0, while the universe passes four inflationary stages. The further growth of t from 0 to 4 results in decrease of the number of interactions to only one, and it is possible to speak on four "deflationary" stages. The initial stage of the universe with the unique $(0,4)$-interaction is not identical to its final stage with the unique $(4,0)$-interaction (both the topologies of the admissible spatial sections and the coupling constants values are different). We conclude that in our model contains the unification of fundamental interactions (known from the gauge theories). It manifests itself here as alternation of the families of interactions; for example, in the transition from the family $\mathcal{M}(-1)$ to $\mathcal{M}(0)$, the number of interactions grows from 4 to 5, and in the latter family they are identified by the values of coupling constants (*cf.* the column $t = 0$ in Table III with the Table I). For any of these low-energy interactions its natural counterparts can be traced both when the quantum number l remains the same, and when n does not change. *E.g.*, the electromagnetic, $(1,1)$-, interaction has four "closest counterparts" in the higher-energy ensembles $\mathcal{M}(\pm 1)$: for $l = 1$ these are $(0,1)$- and $(2,1)$-interactions, and for $n = 1$, $(1,0)$- and $(1,2)$-interactions (see values of their constants in the Table III). Thus we come to a somewhat different unification approach to fundamental interactions than that known in the gauge models. The further study of $(1,1)$- (electromagnetic) interaction shows that its constant after the inflation which describes the transition from the family $\mathcal{M}(-1)$ to $\mathcal{M}(0)$ changes (due

to growth of number of the homological spheres $\Sigma_1^{(1)}$) from $\alpha^{(1)}_{1\ \max} \simeq 1/84.981$ to $\alpha^{(1)}_{1\ \min} \simeq 1/190.219$, see the relation (7). At present the fine structure constant has in our universe the value $\alpha \simeq 1/137.036$. It is remarkable that $\alpha^{(1)}_{1\ \min} < \alpha < \alpha^{(1)}_{1\ \max}$; moreover, a more profound observation suggests itself. The most probable value of α (at the equilibrium) calculated according to (8), is $\alpha_1^{(1)} = 2\pi/\sqrt{V_1^{(1)}} \simeq 1/147.323$. Since the discrete volume of the universe at the equilibrium is greater than it is at present ($\alpha_1^{(1)} < \alpha$), one has to conclude that at the present stage the universe should be expanding, just as it is observed. Strictly speaking, this observation does pertain to the "electromagnetic", *i.e.* $(1,1)$-universe, but it is exactly the electromagnetic radiation which allows us to detect the Hubble expansion effect. Using other low energy interactions, it would be more difficult to argue in this respect, since only the fine structure constant is determined unambiguously and other dimensionless coupling constants involve in the existing theory certain rather arbitrary parameters.

Our approach also makes it possible to evaluate the size of universe as well as the time scales of the stages of its evolution. To this end note that the counterparts of the cosmological $(4,4)$-interaction are the $(n,4)$-interactions ($n \in \overline{0,4}$, $l = 4$) and $(4,l)$-interactions (($l \in \overline{0,4}$, $n = 4$). It is natural to admit that the linear size of (n,l)-universes can be evaluated by the formula $R_n^{(l)} \sim \sqrt[4]{V_n^{(l)}}$. This linear size is dimensionless. To properly normalize it, admit that at the level of the unification of four interactions (strong, electromagnetic, weak, and gravitational ones), *i.e.* for $t = n - l = -3$ ($l = 4$, $n = 1$), the universe should be of Planckian scales. Then $r_1^{(4)} = k\sqrt[4]{V_1^{(4)}} = 1Pl = 1.63 \times 10^{-33} cm$. Admitting the coefficient k to be universal, we evaluate the linear size of the (n,l)-universes as $r_n^{(l)} = kR_n^{(l)}$. Just this quantity (in *cm*) is given in the first line of the Table III. Note that since our model takes into account the unification of five interactions (including the cosmological one), it treats also sub-Planckian scales; moreover, it predicts the radius of universe to be almost ten orders of magnitude larger than that of the observable part of the universe, $R \sim 10^{28} cm$.

Finally, let us summarize the principal results which we came at in this paper.

• A model of universe is constructed where coupling constants values $\alpha_n^{(l)}$ of the fundamental interactions are determined by topology fluctuations in the framework of finite topology ensembles $\mathcal{M}_n^{(l)}$.

• The hierarchy of the coupling constants being obtained in the framework of the topology ensembles $\mathcal{M}(0)$ well reproduces the experimentally known hierarchy of the low-energy dimensionless coupling constants.

• The model describes four inflationary stages which result in the changes of families of the fundamental (n,l)-interactions as well as of the linear scales of the universe (evaluated via the "cosmological" $(n,4)$- and $(4,l)$-interactions). Four "deflationary" stages are also predicted which result for the universe in coming to the final state with only one fundamental $(4,0)$-interaction not being equivalent to the initial one with the $(0,4)$-interaction.

• All the fundamental coupling constants corresponding to the most probable states, are explicitly calculated from the hypothesis that $\alpha_n^{(l)} := 2\pi/\sqrt{V_n^{(l)}}$ where $V_n^{(l)}$ is the mean discrete volume of the T_0-discrete spaces realized as nerves of chc-triangulations of the spaces of the ensemble $\mathcal{M}_n^{(l)}$. These triangulations are compatible with the Milnor representation of the Seifert homology spheres of which are built all the spaces of all ensembles $\mathcal{M}_n^{(l)}$.

• From the known value of the fine structure constant in comparison with that calculated in our model, we explain the universe expansion at the modern stage of its evolution.

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